Limits of slow light in photonic crystals

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While ideal photonic crystals would support modes with a vanishing group velocity, state-of-the-art structures have still only provided a slow down by roughly two orders of magnitude. We find that the induced density of states caused by lifetime broadening of the electromagnetic modes results in the group velocity acquiring a finite value above zero at the band-gap edges while attaining superluminal values within the band gap. Simple scalings of the minimum and maximum group velocities with the imaginary part of the dielectric function or, equivalently, the linewidth of the broadened states are presented. The results obtained are entirely general and may be applied to any effect which results in a broadening of the electromagnetic states, such as loss, disorder, and finite-size effects. This significantly limits the reduction in group velocity attainable via photonic crystals.

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The interest in slow-light phenomena has increased significantly in recent years due to the potential applications in areas such as optical processing,[1–4] quantum information processing,[5,6] enhanced spontaneous emission,[7] and sensing.[8,9] Strongly dispersive periodic structures with dielectric functions that vary on the length scale of the wavelength of light can now be fabricated with impressive precision and resolution.[10,11] By exploiting the close analogies with condensed-matter theory, these so-called photonic crystals can give rise to such optical phenomena as, e.g., enhanced Raman scattering,[12] increased stimulated emission,[7] superprism behavior,[13] and negative refraction.[14] Due to their highly dispersive nature, photonic crystals have emerged as excellent candidates for sources of slow-light phenomena.[15–18] However, even though ideal structures would in principle support modes of vanishing group velocity, state-of-the-art structures have still only provided a slow down by roughly two orders of magnitude.[15,18] The limits imposed on the minimum attainable group velocity in photonic crystals have been studied in various contexts, such as, e.g., fabrication disorder,[19,20] lossy dielectrics,[21] and finite-size effects.[22] It is the aim of this Brief Report to generalize these findings and to show that they may all be presented in the context of broadening of electromagnetic modes and the resulting induced density of states (DOS).

The existence of photonic band gaps (PBGs) in photonic crystals is accompanied by van Hove singularities in the DOS near the band-gap edge, which results in a vanishing group velocity. In one-dimension (1D) the group velocity is inversely proportional to the density of states, while in higher-dimensional structures it is inversely proportional to the projected one-dimensional density of states along the propagation direction, as in mesoscopic electron transport. A formal proof can be given in terms of Wigner-Smith group delay formalism, thus also applying to inhomogeneous structures of finite extension (see, e.g., Ref. 23 and references therein). Letting g_0 denote the one-dimensional projected density of states along the propagation direction of a homogeneous material with a dielectric function given as the average value of the dielectric function of the photonic crystal, we identify three different regimes of interest: (i) a long-wavelength regime, where the properties of the photonic crystal are independent of the detailed geometrical composition, and thus $g(\omega) \simeq g_0$, (ii) a slow-light regime, where $g(\omega) > g_0$, and (iii) a "superluminal" regime, where $g(\omega) < g_0$. We stress that $g(\omega)$ in all cases refers to the one-dimensional projected density of states along the propagation direction and that indeed this is the case whenever we speak of the density of states. Experiments have confirmed the existence of both slow[15–17,24] and superluminal[25] regimes.

Consider the dispersion relation near the edge of a photonic band gap. The exact shape of the dispersion relation naturally depends on the geometry and dimensionality of the photonic crystal. However, common to all band structures is a vanishing slope at the band-gap edge. If we expand the band structure near the band-gap edge $\omega(K) = \omega_0$, we thus obtain

$$\omega(k) \simeq \omega_0 + \alpha(k - \mathrm{K})^2, \tag{1}$$

where α is the group velocity dispersion (GVD). Here we have ignored any higher-order contributions to the expansion. Higher-order dispersion is of vital importance for the propagation of pulses in the slow-light regime,[26] but we ignore it for now as we are only interested in studying the actual value of the group velocity. The group velocity becomes

$$v_g = \mathrm{Re}\left(\frac{\partial \omega}{\partial k}\right) = \mathrm{Re}(2\sqrt{\alpha(\omega - \omega_0)}), \tag{2}$$

illustrating the square-root divergence of the density of states at the band-gap edge. Let us consider the effect of a finite imaginary part of the dielectric function $\epsilon = \epsilon' + i\epsilon''$ of either of the constituents of the photonic crystal. To capture the effect of a small imaginary part we apply standard electromagnetic perturbation theory as described in detail in Ref. 27. The first-order term in a perturbative expansion in the imaginary part of the dielectric function is

$$\Delta\omega = -\frac{\omega}{2}\frac{\langle \mathbf{E}|i\epsilon''|\mathbf{E}\rangle_{\mathcal{V}_1}}{\langle \mathbf{E}|\epsilon'|\mathbf{E}\rangle}, \tag{3}$$

where the integral in the denominator is restricted to the region $\mathcal{V}_1$ of the constituent containing the imaginary dielec-

tric part. The effect is then an imaginary shift of the frequency,

$$\Delta\omega = -\frac{1}{2} i f \omega \epsilon''/\epsilon', \tag{4}$$

where

$$f = \frac{\langle \mathbf{E}|\epsilon'|\mathbf{E}\rangle_{\mathcal{V}_1}}{\langle \mathbf{E}|\epsilon'|\mathbf{E}\rangle}, \tag{5}$$

is the fraction of dielectric energy localized in the corresponding dielectric.[28] Making the substitution $\omega_0 \rightarrow \omega_0 + \Delta\omega_0$ in Eq. (2) means that at the band edge the group velocity becomes

$$v_g = \sqrt{\alpha f \omega_0 \frac{\epsilon''}{\epsilon'}}. \tag{6}$$

Despite its simplicity, this result is very general and independent of the particular photonic crystal geometry, be it of one-, two-, or three-dimensional nature. The result indicates that the vanishing group velocity and the corresponding divergence of the density of states is resolved in the case of a finite imaginary part of the dielectric function. Furthermore, the group velocity has a sublinear dependence on ϵ'' so the reduction in the group index is significant even for small imaginary parts of the dielectric function. In deriving this result we have of course assumed that the slow down is achieved at the band edge, where ideally $v_g=0$. Other schemes exist for achieving slow-light within a flattened band, wherein the group velocity is nonzero.[29] However, including such a term in the previous derivation merely adds an additional term to Eq. (6) inside the square root and as such simply serves to increase the lower limit of the attainable group velocity. Due to the effects on pulse broadening, much attention has been devoted to finding structures with low group velocity dispersion in the slow-light regime.[30,31] Interestingly, our analysis reveals that such structures have the additional benefit of reducing the minimum attainable group velocity. Indeed, in the limit of vanishing GVD our analysis shows that the group velocity attains the ideal value of zero provided that any higher-order dispersion is negligible. Alternatively, we may consider the effect of ϵ'' as a finite broadening of the order $\epsilon''\omega$ of the electromagnetic modes.[27] Thus, any effect causing such lifetime broadening of the modes is subject to a similar analysis as just described and will thus increase the minimum group velocity as the square root of the finite linewidth of the broadened modes. This very general analysis applies to any photonic crystal geometry, and any effect of lifetime broadening, such as, e.g., loss, disorder, or finite-size effects. Of course, the analysis is based on a perturbative approach and as such is only applicable in the case of small perturbations of a perfect photonic crystal. In the limit of strong disorder, for example, Anderson location will significantly alter the delay-time statistics of the photonic crystal.[32,33] In this analysis we limit ourselves to the case where the mean-free path is sufficiently long so that the concept of group velocity is meaningful. In passing, we note that another effect of the broadening of the modes is that a density of states is induced within the band gap. The induced density of states midgap is $g_{\mathrm{PBG}} \propto \epsilon''/(\Delta\omega)^2$, where $\Delta\omega$ is the width of the gap.[27] The result of this is that superluminal group velocities are attained within the band gap, wherein the perturbative analysis reveals that $v_g \propto 1/\epsilon''$.

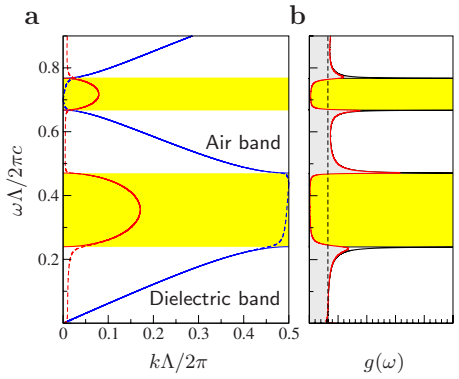

FIG. 1. (Color online) (a) Photonic band structure of the Bragg stack. The real parts of the k vectors are shown in blue, while the imaginary parts are shown in red, with maxima inside the PBGs. The dashed lines indicate the case where the conductivity of the dielectric solid is nonzero. The photonic band gaps are indicated by yellow shading. (b) The corresponding density of states with the case of nonzero conductivity shown in red. The dashed horizontal line indicates the density of states of a homogeneous material with $\epsilon=(a_1\epsilon_1+a_2\epsilon_2')/\Lambda$.

In the following we support these general findings by two illustrative examples, where the source of the broadening is a finite conductivity of the dielectric solid of an air-dielectric structure. We first consider the simple case of a Bragg stack with layers of width $a_1=0.8\Lambda$ and $a_2=0.2\Lambda$ for air and dielectric, respectively, where Λ is the lattice constant. We take $\epsilon_1=1$ for air and assume that the dielectric solid can be described by a frequency independent real part $\epsilon_2'=9$ of the relative dielectric function. The imaginary part is modeled as $\epsilon_2''=\sigma/(\epsilon_0\omega)$, where σ is the conductivity of the dielectric and ϵ_0 is the vacuum permittivity. This is similar to the Drude model for the optical response of metals and is chosen so that the Kramers-Kronig relations are fulfilled.

In Fig. 1 we show the band structure and corresponding density of states of the Bragg stack for the two cases of zero and nonzero conductivity of the dielectric solid, calculated using exact analytical expressions.[34] At zero conductivity we note that two photonic band gaps exist, where there are no states with finite real k values. These PBGs are caused by the complete destructive interference of transmitted and reflected waves at each layer of the Bragg stack. The loss caused by the nonzero conductivity makes the destructive interference incomplete, and the PBGs are no longer fully developed. Consequently, a density of states is induced in the band gap, and the divergence of the density of states at the band-gap edge is resolved. The effect of this is twofold. First, the group velocity at the band edge acquires a finite value larger than zero because now $g \neq \infty$, and second, the group velocity inside the PBG attains superluminal values larger than c because in this region $g < g_0$. We stress that while the group velocity in the superluminal regime may exceed the speed of light in vacuum, this implies no loss of causality but is rather due to strong reshaping of the optical pulse, as discussed, for example, by Büttiker and Washburn.[35]

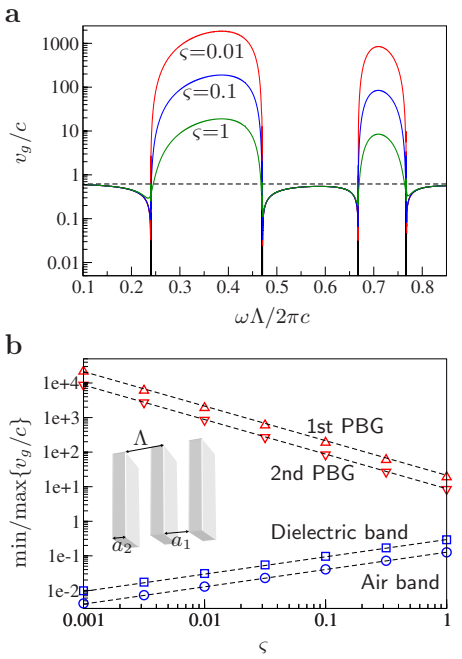

FIG. 2. (Color online) (a) Group velocity as a function of frequency for three different values of the dimensionless parameter $\varsigma = \sigma\Lambda/(2\pi c\epsilon_0)$ characterizing the conductivity of the dielectric solid. The solid black line corresponds to the case of zero conductivity. The dashed horizontal line indicates the group velocity of a homogeneous material with $\epsilon=(a_1\epsilon_1+a_2\epsilon_2')/\Lambda$. (b) The minimum group velocities of the dielectric and air bands of the first PBG, and the maximum group velocities of the first and second PBG. All are shown as a function of the dimensionless conductivity. The dashed black lines indicate fits to the numerical data of the form $A\sqrt{\varsigma}$ for the minimum group velocities and of the form A/ς for the maximum group velocities. The inset shows the geometry of the Bragg stack.

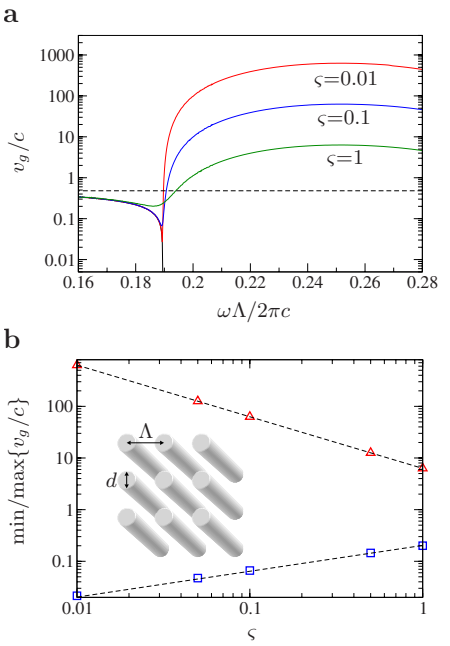

FIG. 3. (Color online) (a) Group velocity as a function of frequency for three different values of the dimensionless parameter $\varsigma = \sigma\Lambda/(2\pi c\epsilon_0)$ characterizing the conductivity of the dielectric solid. The solid black line corresponds to the case of zero conductivity. The dashed horizontal line indicates the group velocity of a homogeneous material with $\epsilon=\epsilon_1+(\epsilon_2'-\epsilon_1)\pi r^2/\Lambda^2$. (b) The minimum and maximum group velocities as a function of the dimensionless conductivity. The dashed black lines indicate fits to the numerical data of the form $A\sqrt{\varsigma}$ for the minimum group velocities and of the form A/ς for the maximum group velocities. The inset shows the geometry of the photonic crystal.

In Fig. 2(a) we show the group velocity as a function of the normalized frequency, for three different values of the dimensionless parameter $\sigma\Lambda/(2\pi c\epsilon_0)$, characterizing the loss of the dielectric solid. To calculate the group velocities we use the definition $v_g=\partial\omega/\partial(\mathrm{Re}[k])$, which yields the same results as the formal definition $v_g=\mathrm{Re}(\partial\omega/\partial k)$.[36] Even for a low value of $\varsigma=0.01$, we find that the group velocity no longer drops below approximately 1/80th of the speed of light in vacuum, and for $\varsigma=0.1$ the minimum group velocity is one-tenth of the speed of light in vacuum. Because of the lower overlap of the electromagnetic field with the dielectric solid in the air bands, the effect of loss on the group velocity is less pronounced for these bands. While the value of the minimum group velocity is clearly highly dependent on the conductivity, the frequency ω_0 at which the group velocity is at its minimum remains nearly constant except for a very slight redshift at high values of the conductivity. In Fig. 2(b) we show the maximum and minimum values of the group velocity as a function of the dimensionless conductivity. The maximum values of the group velocity follow very precisely the $1/\epsilon_2''$ dependence expected from our initial analysis. Assuming a similar distribution of the induced DOS in both band gaps, this analysis also indicates why the maximum group velocity falls off more rapidly in the case of the second narrower band gap. The minimum group velocity is found to follow almost exactly the expected $\sqrt{\epsilon_2''}$ dependence. We note again that the dependence of the minimum group velocity on loss is less pronounced for the air band due to the smaller overlap of the electromagnetic mode with the lossy dielectric. We have verified that these simple scaling laws of the minimum and maximum group velocities hold for any parameters of the Bragg stack and also in the case of oblique incidence.

We next consider a two-dimensional photonic crystal consisting of dielectric cylinders of diameter $d/\Lambda=0.55$ surrounded by air and arranged in a square lattice with lattice constant Λ. We take $\epsilon_2'=(3.88)^2$ for the dielectric cylinders, corresponding to silicon, and model the loss via the Drude model, as for the Bragg stack. We consider normal incidence of TM modes. As for the Bragg stack, the effect of loss is to induce a density of states in the PBGs of the structure. The dispersion relations are calculated using the transfer-matrix method.[37] In Fig. 3(a) we show the group velocity as a function of normalized frequency for increasing values of the dimensionless conductivity. The results shown are for frequencies near the first TM photonic band gap. Similar trends are seen for the 2D photonic crystal as for the Bragg stack, namely, that the density of states induced by the finite conductivity of the dielectric resolves the divergence of the density of states at the band-gap edge, thus causing $v_g>0$ at this point. Also, superluminal group velocities are attained inside

the photonic band gap. In Fig. 3(b) the minimum and maximum group velocities are shown as functions of the dimensionless conductivity. As for the Bragg stack, the maximum and minimum group velocities are approximated very precisely by $1/\epsilon_2''$ and $\sqrt{\epsilon_2''}$ dependencies, respectively, supporting our conclusion that these scalings are indeed universal for any geometry, and even dimensionality, of the photonic crystal.

In this Brief Report, we have for simplicity focused on conductivity as the cause of the smearing of the density of states. However, it is clear that any effect leading to a finite broadening of the electromagnetic states will result in a similar scaling of the minimum attainable group velocity with the order of the broadening. This includes loss, disorder, finite-size effects, etc. and consequently imposes significant limits on the minimum group velocities attainable via photonic crystals. Even in the absence of other broadening mechanisms, absorption will inevitably be present and thus represents the ultimate limiting factor on the attainable reduction in group velocity. In closing, we note that while we have focused our attention to photonic crystals, the analysis presented in this Brief Report applies equally well to any slow-light scheme based on band-structure effects, such as, for example, coupled resonator waveguides, where it has been shown that lattice disorder may severely limit the attainable reduction in group velocity.[38]

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